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Some Metric Considerations in Pattern Recognition

THEODORE A. KALIN

COMPUTER
AND MATHEMATICAL SCIENCES
LABORATORY

ELECTRONICS RESEARCH
DIRECTORATE
AIR FORCE CAMBRIDGE
RESEARCH LABORATORIES
OFFICE OF
AEROSPACE RESEARCH
BEDFORD MASSACHUSETTS

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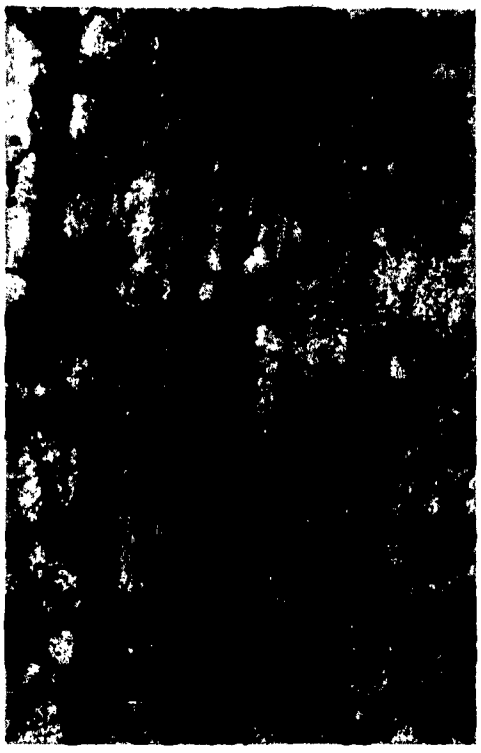
Abstract

The lack of traceable nerve pathways in the visual cortices of mammals has, in the last decade, generated speculation about machines which might learn to recognize visual patterns but which would be randomly wired to their source of information. This report analyzes such devices and shows that at least certain mild constraints of a distance-ordering sort must be imposed upon random connections. Mammalian visual system neuroanatomy is then re-examined in this light. This work was carried out under Project 5632 and Task 56325, and it was submitted for publication in February 1961.



Two neurons from the visual cortex of a cat. The staining technique used affects only a portion of all nerve cells but stains these rather thoroughly; in an actual brain the neurons are packed more closely together than this figure would suggest. The larger branching processes are dendrites concerned with receiving signals from other cells. An output axon is visible on the left as the small fiber descending from the base of the main cell body. (Illustration reproduced from *The Organization of the Cerebral Cortex*, by D. A. Sholl, through courtesy of Methuen & Co. Ltd., Copyright 1956.)

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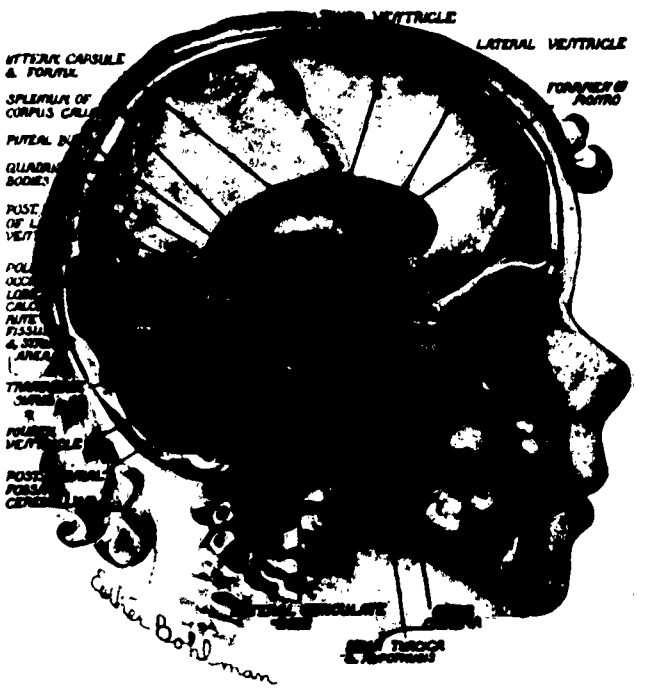


LEFT

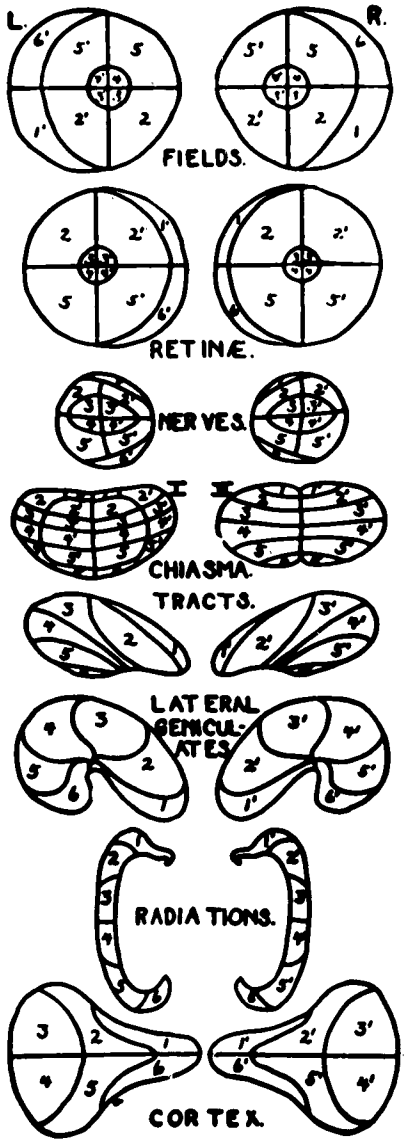
X-ray photograph, lateral view, of head of normal eight-year-old child with the visual system drawn in its normal position. The optic nerve leaves the eyeball at right and divides to send fibers to the opposite geniculate and to receive fibers from the opposite eye at the optic chiasma, about an inch behind the eyeball. Each optic tract (only one of which is indicated) to the left of the chiasma thus contains fibers drawn more or less equally from both eyes, and ends in the lateral geniculate body in the center of the photograph. From here new fibers, the optic radiations, fan out to the visual cortex (striate cortex) at the rear of the brain, where their endings are indicated by heavy dots. (Illustration reproduced from *The Vertebrate Visual System*, by S. Polyak, through courtesy of the University of Chicago Press. Copyright 1957 by the University of Chicago.)

RIGHT

Continuous mapping of the visual field through successive stations in the visual pathways culminating at the striate cortex. For each half of the visual field adjacent regions of the field map eventually on to adjacent regions of the cortex. (Illustration reproduced from *Functional Neuroanatomy*, by W. J. S. Krieg, McGraw-Hill Book Company, 1953, through courtesy of the author.)

Some Metric Considerations In Pattern Recognition

I Introduction 1



In terms of specificity of organization we may conceive of pattern recognizing machines as arranged on a scale from the completely randomly connected at one end to precise configurations capable of responding only to pre-defined characters at the other. The former, of which several have been proposed in the literature, would be expected to acquire a "set" toward useful characteristics of patterns presented to them during a learning phase due to some built-in plastic property of their makeup, while the latter are merely templates responding only to exact copies of patterns initially designed into them. Some machines of this latter kind are now in actual productive operation sensing magnetic ink characters on bank checks and so on, but since their design features are so completely known they tend to lack speculative interest. The random machines, on the other hand, seem to have been arrived at by analogy to living networks of interconnected nerve cells whose apparent lack of traceable circuits — especially in the visual cortex of mammals — is one of the puzzles of neuroanatomy.

Many of the recently published studies on automatic pattern learning refer to devices well in the middle of this scale or toward the "highly organized" end of it, and workers in the art increasingly seem to feel that a strong initial organization of an hierarchical kind is a necessary feature of successful automata which can be taught to recognize patterns.

The puzzle of apparently random cortical connections found in mammalian visual systems remains, however, to suggest fruitful machine analogies toward the unorganized end of the scale. It is the purpose of this report to examine rather rigorously some consequences of random

organization which in turn is shown not to be practical in pure form. Very mild constraints are then placed upon otherwise random connection paths in an attempt to define a most general (that is, non-specific) class of workable machines. These constraints are stated in terms of distance in Euclidean space and, building upon the distance ordering of that space, lead logically to rather highly specific restrictions upon permissible connection modes. A brief survey of recent neurophysiological literature on the visual systems of cats, primates and man indicates that a corresponding specificity is encountered there as well, and that it is more geometrical and morphological than point-to-point.

In terms of the framework developed in this report it seems safe to conclude that machines in the immediate neighborhood of the random end of the scale can legitimately be excluded from future consideration and that plausible phylogenetic reasons exist for the observed continuous mapping of the visual field on to the occipital pole in mammals.

This report is intended for readers in neuroanatomy as well as computer technology. A glossary of terms is accordingly supplied, and the author respectfully requests indulgence in advance for obvious oversimplifications committed in the name of brevity. A less technical version for the general reader will be available as a separate document under the title "Elementary Pattern Perception in Machine and Mammal."

II Frames and Patterns

We will work with binary-quantized information for two important reasons: first, most equipment so far proposed for automatic pattern recognition makes use of digital processes on quantized data, and several digital computer programs have already been employed in related research (Farley and Clark; Selfridge; Rosenblatt; Bledsoe and Browning; Roberts; Doyle); and second, binary quantization makes it easy and profitable to use concepts from elementary combinatorial mathematics and the theory of finite point-sets.

By "pattern" we shall mean something like a meaningful picture. A pattern may be a radar PPI display of an air traffic situation, a time-slice of speech sounds recognizable by a human listener, a single typed or printed alphabetical character, a common geometrical shape on a two-dimensional spatial field, a sequence of Morse code signals to which meaning is assigned by convention, and so on. It will be understood that such patterns are quantized into n bits unless otherwise stated (Shannon). In particular it is convenient to imagine visual patterns binary-quantized in intensity and displayed upon a $\sqrt{n}$ by $\sqrt{n}$-bit surface. Any arbitrary arrangement of n bits will be called a "frame," a term adapted from radar and television usage. Thus all patterns are frames, but most frames are not patterns, the distinction being that patterns are frames which are humanly meaningful in some appropriate sense.

Where one pattern leaves off and another begins is usually clear in a given particular context but is somewhat difficult to tie down in the abstract. In the case of visual frames we may imagine temporal sequences of successive n-bit two-dimensional spatial patterns. In quantized speech, each

pattern might be defined as some rather arbitrary time interval and its associated frequencies and amplitudes, so that sequences of patterns are successions of such intervals. (Indeed, one of the difficulties in speech work seems to be the lack of consistent partitions between adjacent patterns, dependent as these are upon the varying modes of articulation characteristic of the human speech-generating apparatus.) Whatever pattern-defining dimensional scheme is used in some particular instance, however, will also serve to define the corresponding frames, and frames and patterns alike will be assumed quantized into n bits as previously stated.

Problems in automatic pattern recognition usually have as their goal the development of a computer program or of a machine which will give a predetermined response when excited by any member of some class of equisignificant patterns. By "equisignificant" we mean that two or more patterns have the same meaning to the machine's designers and operators (Reichenbach; Selfridge, 1955). The crucial point here is that equisignificance, being a semantic property, is a matter of human decision or of historical convention arising from the evolution of shared experiences, and automatic pattern recognition — even in principle — is possible at all only because the notion of equisignificance is reasonably well-correlated with certain informational properties of the patterns themselves. These properties, in terms of which human beings classify patterns according to equisignificance, in turn depend heavily upon such fundamental ordering concepts of topological and metric spaces as contiguity, continuity, connectivity, and so on, at least in most practically useful instances. Examples of such spaces are Euclidean metric space of two dimensions, appropriate to visual patterns, and the pair of continuous one-dimensional spaces in frequency and intensity often used in speech analysis.

Many machine proposals for pattern recognition involve some self-adaptive or "learning" function, by means of which a machine can be "taught" to respond to a given equisignificance class by presenting to it a sequence of patterns drawn from that class, with or without human monitoring. In the case of quantized patterns it is obvious that temporally adjacent members of such a sequence may have several bits in common; this phenomenon will be referred to as "overlap." Mathematical analysis will be facilitated, however, if we confine our attention to sequences of patterns in which the overlaps are more or less random, so that order in time is not important. Disregarding temporal order, then, we may speak simply of classes of patterns and of frames, and any given sequence defines such a class.

It is clear that there are 2^n distinct frames possible, corresponding to the 2^n ways of assigning values either 0 or 1 to n bits. Further, we may define frame classes by considering sequences of 0, 1, 2, 3, ... frames; a class of k frames can be constructed from the 2^n possible in $\binom{2^n}{k}$ different ways, the number of ways in which 2^n things can be chosen k at a time. Thus where C is the class of frame classes and $N(C)$ is the number of members it contains,

$$N(C) = \sum_{i=0}^{2^n} \binom{2^n}{i}$$
$$= 2^{2^n}.$$

That $N(C)$ is an upper bound is easy to

prove by indirection. Suppose we have an $N(C) + 1^{st}$ class of frames allegedly not to be found in C. This class will have some definite number, say $l < 2^n$, of frames as its members. But in the above enumeration defining $N(C)$ we have already included all possible l-frame classes in the series of $\binom{2^n}{l}$ terms; hence our supposedly new class has already been counted in the enumeration.

III Metrics and Classes on n-Bit Spaces

Consider a two-dimensional $\sqrt{n}$ by $\sqrt{n}$-bit frame whose binary elements are arbitrarily labeled $a, b, c, \ldots$, in some order. If these labels are assigned to the frame elements randomly, as by consulting a table of random numbers or by drawing the labels from a hat as the frame elements are systematically scanned, then no order of label assignation can be called "preferred" over any other order. In particular, any two labels may be transposed without affecting the randomness of the labeling, and any arrangement of labels can be generated from any other by a finite sequence of pairwise interchanges. What meaning can be given to the notion of "distance" between pairs of points randomly labeled in this fashion?

It is commonly agreed (Busemann) that any distance-defining function, or metric, on a space of n points must meet at least the following four conditions: where $d(x, y)$ is such a metric and x, y, z, are any three of the n points, $d(x, y)$ defines a real number satisfying

I. $d(x, y) > 0, x \neq y$

II. $d(x, x) = 0$

III. $d(x, y) = d(y, x)$

IV. $d(x, y) + d(y, z) \geq d(x, z)$.

But we have just seen that for the random n-bit frame we may arbitrarily transpose the labels of any two elements; hence any acceptable metric must also be invariant under transposition. The only metric meeting this condition, as well as I through IV above, is

$d(x, y)$ = constant, for all $x \neq y$.

We prove this by assuming its contradiction: suppose $d'(x, y)$ is some proposed metric such that, for some x, y, z,

$$d'(x, y) \neq d'(x, z).$$

Then either

$$d'(x, y) > d'(x, z) \text{ or } d'(x, y) < d'(x, z)$$

but not both. Regardless of which of these two inequalities is assumed true it can be falsified by transposing y and z. Hence

$$d'(x, y) = d'(x, z),$$

equivalent to stating that the distance between any two non-identical points is constant, and their distances from one another cannot be ordered save in a trivial sense.

On the random n-bit frame, then, we have no way of adequately taking into account order- and metrically-derived properties of patterns other than identity or non-identity of bits. In particular we have no way of describing the following humanly important properties of points a, b, c, . . . , comprising patterns:

1. Point a is farther from b than from c.
2. a, b, c, d, . . . are the only points equidistant from p.
3. a is adjacent to b (that is, there is no point c such that $d(a, c) < d(a, b)$).
4. Points (a_i) are such that a_i is adjacent to a_{i+1}, so that a_k to a_l, $k \leq i < l$, form a continuous "line".
5. Points (a_i), $k < i < l$, describe a simply-connected closed figure. That is, only a_{i-1} and a_{i+1} are adjacent to a_i, $a_k = a_l$, and no other $a_i = a_j$, $i \neq j$, $k < i$, $j < l$.

The reasoning behind the preceding argument can be put another way. Let us imagine a storeroom filled with all possible n-bit random frames, from which one is blindly chosen. After some examination, suppose we assert that we can define a metric $d'(x, y)$ in terms of which $d'(x, y) > d'(x, z)$ for some triples of points on that frame. Then there exists in the storeroom at least one other frame for which $d'(x, y) < d'(x, z)$ for some of the x, y, z's in question, namely that frame generable from the original by appropriate interchanges of y's and z's. Since that second frame might just as well have been chosen initially, we must conclude that the only metric holding for all frames is one which defines some constant distance between all non-identical x's and y's. Thus again we have no way of describing adjacency, connectivity, and other related properties of patterns, and we must conclude that only non-ordered (or trivial, such as overlapping) properties can meaningfully be treated in the case of random frames.

The term "random automaton" is introduced here in a loosely-defined way to refer to any pattern recognizing learning machine whose input is derived from a random frame, that is, whose initial structure or organization is essentially unaffected by permuting elements of the input frame. Such machines have been discussed by Sholl and Uttley; Uttley;

Rosenblatt; Day and Newman; Minsky and Selfridge. It is especially interesting to consider the behavior of these devices after excitation by a sequence of patterns impinging upon the input elements with no concomitant reinforcing and inhibiting signals from the experimenter.

We have seen that a useful metric cannot be defined for such a system because, being random, the initial connections between input elements and the rest of the device can be arbitrarily permuted. Consider now two such automata differing only in some permutation P of the input elements of the second with respect to those of the first. Both are random automata, and both will respond identically (we here presuppose no internal noise source) if the first is exposed to some n-bit pattern and the second to that same pattern permuted by P. Yet P may be any permutation, and may transform the pattern into what appears to be a randomly-spaced collection of 1's and 0's on the n-bit frame. Conversely, there exists a permutation P' which transforms an arbitrary arrangement of bits into some prespecified pattern having the same proportion of 1's and 0's. We cannot reasonably expect, then, that a random automaton will be biased toward responding to (humanly-important) patterns to a greater degree than toward random arrays of bits, provided the temporal distribution of overlaps is also approximately random in both types of sequences. In particular, there is no reason *a priori* to expect response discrimination in favor of some one property with respect to which all patterns in a sequence are equisignificant.

Yet if the random automaton has certain characteristics of instability, so that it tends to favor responses toward some characteristic of a sequence of patterns presented to it as the sequence proceeds, how can we evaluate the chances of a given property being singled out for such response reinforcement? Better stated, after t patterns have been "shown" to the device, how can we predict which of the remaining $2^n - t$ frames will elicit a positive response from it? To do this we clearly must form an idea of the maximum number of possible ways of categorizing frames and some estimation of their relative probabilities. It has already been shown that classes of patterns defined according to metric or ordering properties can expect no preferential treatment in a random automaton, and in Section II we derived the result that there are

$$N(C) = 2^{2^n}$$

distinct possible abstract classes of n-bit frames. Any sequence of 2^n or less distinct patterns must fall into one or another of these classes (disregarding time order as before, on the assumption of random time-distributions of overlaps), and in fact a sequence of $t < 2^n$ distinct patterns will find membership in

$$2^{2^n - t}$$

classes simultaneously. This is proved by the fact that once t of the possible 2^n frames have been employed there remain $2^n - t$ of them each of which may be included or not in a total of

$$2^{2^n - t}$$

classes, and a like number of new classes may be generated by the class sum of these

with that containing the original t patterns. Under these conditions it is useful to attempt to estimate the probability that a random automaton will select the class of equisignificant patterns which the operator has in mind, from the set of possible classes also containing the same patterns.

Let E be a class of patterns equisignificant to the experimenter because all share some predefined property recognizable by him. We specify further that E is well-defined in the sense that the experimenter can always decide unambiguously whether or not any given frame belongs in E, and that E contains all patterns which share the given criterion of similarity. Obviously, for E to be useful,

$$N(E) < 2^n.$$

In addition it may easily turn out that

$$N(E) > t,$$

in which case there are

$$\binom{N(E)}{t}$$

distinct pattern classes (distinct t-term sequences disregarding order) formable from the members of E. The random automaton is to build up response behavior to some class of patterns, preferably to E, so that it hopefully will give an unambiguous positive response only when shown a $t + 1^{st}$ member of E. What are the chances of its accomplishing this? We have noted that

$$2^{2^n - t}$$

members of C contain all t patterns and that only one of these classes is E, but lack of mutual exclusion complicates matters. If we consider those classes having $t + 1, t + 2, \ldots, t + k$ frames as members and also including the initial t patterns, there are

$$2^n - t, (2^n - t)(2^n - t - 1), \ldots$$

$$\ldots, \prod_{i=0}^{k-1} (2^n - t - i) = \frac{(2^n - t)!}{(2^n - t - k)!}$$

$$\cong 2^{kn} \text{ for } 2^n >> t + k$$

classes respectively containing these patterns, the ones to the right including all those to their left. Let us consider the first of these categories only, the $2^n - t$ classes each of $t + 1$ members, of which t are found in E, and let us assume for the moment that E has precisely $t + 1$ members. Since our machine is random in the sense already discussed it seems reasonable to assume that each of these $2^n - t$ classes is close to being equiprobable, that is, that the chances are roughly the same that it will have organized itself to any one of the $2^n - t$ possible classes containing t members plus one more frame. Hence it appears that its chances of responding favorably to the $t + 1^{st}$ member of E is about

$$(2^n - t)^{-1}.$$

If there were two patterns remaining in E after the first t of them had been presented to the automaton, the corresponding chance of proper response to the remaining members of E would be about

$$((2^n - t)(2^n - t - 1))^{-1},$$

and so on, the probability decreasing by the factor $(2^n - t - k)^{-1}$ as we go from the k^{th} to the $k + 1^{st}$ pattern in E after the first t of them. In any event it is clear that these probabilities are quite small unless t is rather close to 2^n. But for the machine to be useful in any real sense t should be very substantially less than 2^n or else there will not be much capacity for new "experience" left in the device. For a rather minimum frame structure of 25 by 25 bits $2^n = 2^{625}$, and it is clear that no actual sequence of stimuli can approach such a value in number of terms, thus effectively meeting this last condition. But by the same phenomenon the chance of the machine correctly responding only to a $t + 1^{st}$ pattern in E is almost vanishingly small.

The preceding discussion assumed that all of the equisignificant patterns comprising E were known, in the sense that an exhaustive enumeration was possible. We now turn to classification of patterns in terms of some similarity property or properties shared by them, rather than in terms of a complete enumeration of the members of each class. As before, however, our objective is to estimate the number of such classes which can exist and hence to form a rough guess as to the chances of a random pattern-learning automaton's adapting itself to respond only to patterns of one given class. It is again assumed that no human monitoring of the device occurs as the trials proceed. Suppose some property, p, is "squareness," and we stimulate a random automaton with a sequence $S = S_1, S_2, S_3, \ldots, S_t$ of frames, each one of which is a square pattern or not, making no attempt to enumerate the total possible number of square patterns. Clearly the members of S can be grouped into two classes (assuming random overlapping as before) according to whether or not their members are square patterns. In conventional set-theoretic notation these two classes are

$$\{x \mid px\},$$

the class of all x's which have the property p (it being assumed that x is in S), and

$$\{x \mid p'x\},$$

the class of all x's in S which do not have the property p. We now introduce another property, q, not related to p, such as thick or thin lined figures — that is $\{x \mid qx\}$ is the class of all thick-lined figures in S and $\{x \mid q'x\}$ is the class of all thin-lined figures, every pattern in S being either a thick-lined or a thin-lined figure. In terms of both p and q it is clear that we can define four disjoint subclasses of S as follows:

$$c_0 = \{x \mid p'x \cdot q'x\},$$

the class of all x's in S which do not have the property p and do not have the property q — that is, which are neither square or thick-lined figures;

$$c_1 = \{x \mid p'x \cdot qx\},$$

$$c_2 = \{x \mid px \cdot q'x\}, \text{ and}$$

$$c_3 = \{x \mid px \cdot qx\}.$$

Now these four disjoint subclasses can be logically summed in sixteen distinct but not necessarily mutually exclusive classes which categorize the members of S in terms of the two properties p and q (in

the notation to follow "+" is read "or," just as "·" stands for "and"):

C_0 = null class

$C_1 = c_0 = \{x \mid p'x \cdot q'x\}$

$C_2 = c_1 = \{x \mid p'x \cdot qx\}$

$C_3 = c_2 = \{x \mid px \cdot q'x\}$

$C_4 = c_3 = \{x \mid px \cdot qx\}$

$C_5 = c_0 \cup c_1 = \{x \mid p'x\}$

$C_6 = c_0 \cup c_2 = \{x \mid q'x\}$

$C_7 = c_0 \cup c_3 = \{x \mid p'x \cdot q'x + px \cdot qx\}$

$C_8 = c_1 \cup c_2 = \{x \mid p'x \cdot qx + px \cdot q'x\}$

$C_9 = c_1 \cup c_3 = \{x \mid qx\}$

$C_{10} = c_2 \cup c_3 = \{x \mid px\}$

$C_{11} = c_0 \cup c_1 \cup c_2 = \{x \mid p'x + q'x\}$

$C_{12} = c_0 \cup c_1 \cup c_3 = \{x \mid p'x + qx\}$

$C_{13} = c_0 \cup c_2 \cup c_3 = \{x \mid px + q'x\}$

$C_{14} = c_1 \cup c_2 \cup c_3 = \{x \mid px + qx\}$

$C_{15} = c_0 \cup c_1 \cup c_2 \cup c_3 = S.$

If our automaton has the capability of organizing itself to respond to patterns of just one class after t trials, as would be the case if it has but one binary output indicator, then even if it were somehow restricted to classifications only in terms of p and q it is far from obvious which of the above sixteen classes would be selected. On a random basis and assuming that c_0 through c_3 have about the same number of members we might estimate that C_{15} would be the most favored class, followed by C_{11} through C_{14}, and so on.

Introduction of a third property, r, would have expanded the number of the C_i to 256, and so forth, a process which approaches but cannot exceed n properties since $N(C)$ cannot be greater than 2^{2^n} as was proved in Section II.

The present analysis of pattern learning behavior in terms of properties also differs from our earlier treatment of enumerated classes secondarily in the makeup of S. In the earlier treatment we dealt with a class E of equisignificant patterns which was included in S but made no comment about those frames in S which were not in E other than to require that E be well-defined. In the context of the present discussion, on the other hand, E would be but one of several classes (the C_i) defined by the properties of its members with respect to which they are equisignificant. Taken collectively the C_i exhaust S; reference to the list at the left will verify that

$$C_i \cup C_{15-i} = S,$$

and in general it can be shown that

$$C_i \cup C_{2^{2^k}-1-i} = S$$

for k properties, provided the listing is carried out as illustrated at the left. Expressions of the form

$$C_i \cup C_{\omega-i} = S$$

also refer to "self-dichotomizing" behavior in terms of a single property p ($\omega = 3$),

and by implication in terms of additional properties $q, r, \ldots$; that is, partitioning of response behavior without regard to the sign of the output binary response.

Both the earlier and the present approach have shown that a large plurality of possible classifications exist potentially in addition to that one to which the experimenter would have his random learning machine adapt itself, and hence that adaptation is much more likely than not to occur with respect to some undesired pattern category unless he actively interferes with the process. Further, in the case of external reinforcement of desired adaptive behavior and inhibition of undesired behavior these analyses suggest that the experimenter may be forced to wait an excessively long time for favorable responses to appear if the machine is somehow modified by chance between trials. We will therefore find it profitable to examine some simple constraints upon an otherwise random automaton which favor machine classification of patterns in terms of distance measure.

IV Mappings

Whether a pattern-recognizing learning automaton is operated in a spontaneous-learning mode or is reinforced and inhibited by a human operator who decides whether its performance at each trial is acceptable or not, a considerable gain in effectiveness may be obtained if machine organization is arranged to take advantage of order- and metric-derived properties of patterns such as those listed on page 8. One would then expect that time wasted by undesirable responses could be reduced substantially. In fact, an important topic beyond the scope of this paper has to do with optimum degrees of organization initially designed into learning machines; by analogy with biological organisms there is reason to suspect that considerable initial organization of a functionally hierarchical kind has strong advantages, and many learning machines have been designed with this in view. In the brief discussion which follows, however, we will take but a short step from the random automata treated earlier, and will examine some aspects of including a distance-measuring capability in terms of which metrically derived properties of patterns can have machine correlates. We therefore abandon random frames to consider only those in which arbitrary transposition of frame bits prior to learning activity is not admissible. Our restrictions upon frame bit interchangeability will be stated in terms of two simple and rather minimal constraints upon effects produced by simultaneously stimulated frame bits in terms of their mutual spacing, and upon effects produced by activated machine elements as a function of physical distances between them. We will then be in a position to discuss preferred mappings or connection paths from input frame to a set of machine elements.

The first constraint (C_1) to be adopted is that pairs of frame bits simultaneously stimulated shall, other things being equal, tend to have more influence upon machine organization and behavior the closer together they lie in the input frame, and equal influences for equal separations. By "tend to have" we shall mean that C_1 is a constraint upon an otherwise random probability distribution with respect to location of bits in the input frame. One would expect, in a not completely homogeneous machine, that some of its parts will have greater effect upon its behavior and organization than others; C_1 merely requires us to associate such parts or elements more frequently with close frame bits than with more widely separated ones according to some properly monotonic inverse function of their separation, for the avowed purpose of emphasizing organization of machine functional substructures in response to adjacent and quite close pattern bits. These can then (a topic not pursued in this paper) serve as building blocks for recognizing patterns designed according to metrically-important schemes, a category including practically all patterns for which machine-learned recognition is desirable.

Since we deal with pairs x, y of input frame bits, and have been speaking of tendencies toward organizational changes rather than rigidly determined ones (although this may be required merely by our incomplete knowledge of the machine's structure), constraint C_1 lends itself to easy expression in terms of conditional probabilities. Let $f(x, t)$ be some organizational change which may occur at the t^{th} learning trial depending upon the value, 1 or 0, of x, and let $p_f(x, t)$ be the probability that $f(x, t)$ actually does occur for $x = 1$. (We omit consideration of $x = 0$ except to note that the functional forms in t of $f(1, t)$ and $f(0, t)$ need not be simply related to each other and assume merely that they are not so antagonistic as to vitiate C_1 over many trials.) Then before C_1 is imposed $p_f(x, 1) = p_f(y, 1)$ for all x, y since nothing can be said *a priori* about preferential effects of stimulated bits in the input frame of a random automaton at the first trial. If we then assume as in Section III that successive patterns as $t = 1, 2, 3, \ldots$ have the same proportion of 1's and 0's and that overlaps are random as well,

$$p_f(x, t) = p_f(y, t), \text{ and}$$

$$p_f(x \mid y) = p_f(f(x, t) \mid y = 1),$$

the probability that $f(x, t)$ will occur at the t^{th} trial given $y = 1$, is symmetric in x and y and quite independent of the location ($x \neq y$) of these two stimulated input bits. Now C_1 merely implies that $p_f(x \mid y)$ will increase as x and y draw closer together:

$d_e(x, y) \leq d_e(w, z)$ implies

$$p_f(x \mid y) \geq p_f(w \mid z) \qquad (C_1)$$

where equalities and inequalities hold together, x may or may not be identified with w, and d_e is distance in a Euclidean space E^2 of two dimensions (the input frame).

Of all ways of implementing this requirement perhaps that involving the least stringent assumptions *a priori* as to structural configuration can be made to follow from a similar constraint (C_2) upon machine elements themselves: pairs of such

elements, when simultaneously stimulated by signals from the input frame, shall tend to have more influence upon machine organization the closer together they lie in the machine, and equal separations shall, by and large, imply equal influences. Thus

$d_e(x, y) \leq d_e(w, z)$ implies

$$p_f(x \mid y) \geq p_f(w \mid z) \qquad (C_2)$$

where equalities and inequalities hold together, x, y, w, z are machine elements connected to the input frame and x may or may not be identified with w, $f(x, t)$ is some function describing a change in machine organization which may occur for $x = 1$, that is, x receives a stimulus from some $x = 1$ in the input frame, and $p_f(x \mid y)$ is the probability of its occurrence given $y = 1$, $p_f(x \mid y) = p_f(y \mid x)$ for all machine elements x, y connected to the input frame, and d_e is Euclidean distance in three dimensions (a physically realized machine occupies a volume in space E^3).

The constraints have been stated without reference to particular automata. In the case of the conditional probability pattern learning machine of A. M. Uttley (1959), C_1 would require the coincidence of $x = 1$ with $y = 1$ to be given greater weight, in a single "and" component eventually accepting inputs from both x and y, the closer together x and y lie in the input raster of photocells. In the "perceptron" proposed by F. Rosenblatt, a similar comment would apply to retinal input units and common association units, and C_2 would imply that such a requirement could perhaps be implemented by specifying that any otherwise random interconnections between association units be more likely to occur the closer such units lie to each other in the machine proper. In any event, in this paper C_2 is not dependent upon C_1 but is a distinct and separate constraint, and is adopted here to avoid commitment to some particular and possibly rather highly structured machine organization complying with C_1.

We have made no comment as to the number of machine elements compared with the number of input frame bits, nor about what kind of connection schemes are to be preferred in mapping the latter on to the former. The existence of C_1 and C_2 implies some rather specific statements about such matters but these further restrictions can be made to follow deductively from C_1 and C_2 together with a few rather obvious assumptions.

Let X be the set of input frame bits $x, y, \ldots, z$, $N(X) = n$, and $\sqrt{n}$ an integer for a $\sqrt{n}$ by $\sqrt{n}$ array in E^2. X is thus a finite metric space. Then (Lemma 1) for $n > 4$, for every pair x, y in X there is at least one z in X such that $d_e(x, z) \neq d_e(y, z)$. Proof: since X is a Euclidean 2-space the locus of points equidistant from x and y is a straight line linearly relating the two components of any elements in X which lie upon it. There can be no more than $\sqrt{n}$ such points, and at least $n - \sqrt{n} - 2$ remain other than x and y.

We will denote by M the set of machine elements and by X that subset of them connected to the input frame as specified by some mapping φ of X on to X, $X \subset M$. It is assumed that $n > 4$ in all that follows, and for the moment we shall exclude any one-many "mappings" of X on to X. Then —

THEOREM 1: X has at least n elements.

Proof: if not, at least two members x, y of X map on to one $\boldsymbol{x}$ in $\boldsymbol{X}$. By the lemma we can find a z in X not equidistant from x and y, and which maps into $\boldsymbol{X}$ by definition. Then either $\varphi z = \boldsymbol{x}$ or not. If so, three points map on to one, and alternative stimulations by differently separated pairs (x, y) and (x, z) exert indistinguishable effects, contradicting $\boldsymbol{C}_1$. If $\varphi z = \boldsymbol{z} \neq \boldsymbol{x}$ then $\varphi(x, z) = \varphi(y, z) = (\boldsymbol{x}, \boldsymbol{z})$ and a similar comment applies. Hence $N(\boldsymbol{X})$ is not less than n.

COROLLARY: $\boldsymbol{M}$ has at least n elements.

COROLLARY: No φ is many-one.

COROLLARY: No $\boldsymbol{x}$ in $\boldsymbol{X}$ has more than one image in X.

COROLLARY: φ is one-one, since one-many "mappings" are excluded arbitrarily.

COROLLARY: $\boldsymbol{X}$ has exactly n elements.

LEMMA 2: Since φ is one-one, $d_e(x, y) \leq d_e(x, z)$ implies $\boldsymbol{d}_e(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{d}_e(\boldsymbol{x}, \boldsymbol{z})$, equalities and inequalities holding together. Proof: this lemma follows directly from $\boldsymbol{C}$ and $\boldsymbol{C}$. Since $\varphi(x, y, z) = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ uniquely, the only way closer points in X can influence the machine to a greater extent than more distant points is through mapping respectively in to closer and more distant machine elements. Important further consequences of this lemma will be discussed later.

THEOREM 2: $\varphi X = \boldsymbol{X}$ maps closest points into closest points. Proof: let x, y be closest: then of course there is no point closer than x to y in X. Suppose a $\boldsymbol{w}$ exists in $\boldsymbol{X}$ such that $\boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{w}) < \boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{x})$. Its image w in X can then not be closer than x to y. CASE I: $d_e(y, w) > d_e(y, x)$. This cannot imply $\boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{w}) < \boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{x})$ by Lemma 2. Thus $d_e(y, w) \leq d_e(y, x)$ apparently remain as possibilities. CASE II: $d_e(y, w) = d_e(y, x)$. This must imply $\boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{w}) = \boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{x})$ by Lemma 2 and hence cannot imply $\boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{w}) < \boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{x})$. CASE III: $d_e(y, w) < d_e(y, x)$. This, the only case which can imply $\boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{w}) < \boldsymbol{d}_e(\boldsymbol{y}, \boldsymbol{x})$, is explicitly forbidden by hypothesis. Hence $\boldsymbol{x}$, $\boldsymbol{y}$ are closest points in $\boldsymbol{X}$.

COROLLARY: If $x, y, \ldots, z$ are closest to w in X then $\varphi(w, x, y, \ldots, z) = (\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{y}, \ldots, \boldsymbol{z})$ in $\boldsymbol{X}$ such that $x, y, \ldots, z$ and $\boldsymbol{x}, \boldsymbol{y}, \ldots, \boldsymbol{z}$ are respectively equidistant from w and $\boldsymbol{w}$.

This theorem and its corollary form a finite-space analogy to the mapping of neighborhoods on to neighborhoods in infinite spaces with completely ordered distances, and hence suggest the notion of continuous mappings. A moment's reflection will show, however, that the analogy is far from perfect in the present context, and that many properties of topologically-invariant mappings on infinite spaces do not hold. For example, we must not allow excessive bending or other "continuous" distortions which would violate Lemma 2 by upsetting the order of any triplets. In the case of infinite spaces continuous mappings admit violent deformations such as stretching, warping, squeezing, and so on, and in fact all distortions not involving tearing or joining, because neighborhoods and their maps may be arbitrarily small. But where, as in this discussion, there is a lower bound to the size of a "neighborhood" (a set of closest points in

a finite metric space) the permissible distortions generable by φ may well be so gradual as to be almost imperceptible, especially when Lemma 2 is kept in mind. Were we certain that X itself formed a (perhaps warped) surface in E^3 of some useful kind, the question would be somewhat clarified, but this remains to be shown.

DEFINITION: A finite set of points in M is a "surface" if the set is embedded in a bounded infinite set which is a two-dimensional space lying within a volume in E^3 also containing M.

This definition, while intuitively reasonable, is not alone sufficient to establish useful conclusions about surfaces in M containing $\boldsymbol{X}$. To illustrate, since $N(\boldsymbol{X}) = n < \infty$ we can always pass $m \leq n$ parallel planes through M in such a way that any member of $\boldsymbol{X}$ lies on one of them. Alternative joining of adjacent planes on opposite sides of the volume containing M then generates one folded planar surface containing $\boldsymbol{X}$. Thus, while $\boldsymbol{X}$ always lies in *some* surface in M, we shall be more interested in those bearing more useful relationships to the E^2 containing X. For example, it might be of some interest to consider surfaces on which closest points in X map as closest points, distances on the surface being measured along geodesics, or shortest curves, between points. Such an approach would not only involve us in rather formidable geometric difficulties, however, but would make use only indirectly if at all of one of our basic assumptions — that distances in M are to be measured by the Euclidean metric in three dimensions. We shall therefore attempt to derive characteristics of surfaces containing $\boldsymbol{X}$ which are "simplest" in some reasonable sense by building upon properties already assumed or deduced and will start this process with consideration of lines and curves in X and $\boldsymbol{X}$. It is clear that the notion of a "curve" in X or $\boldsymbol{X}$ is related to that of a surface, but is of dimensionality one less. We shall confine our attention to curves built up of straight-line segments joining adjacent points in X, or in $\boldsymbol{X}$, or, almost equivalently, as consisting of sequences of adjoining bits in X or in $\boldsymbol{X}$.

DEFINITION: A monotonic curve in X is a sequence (x_i) in X, $i = 1, 2, 3, \ldots, l$, for which $d_e(x_i, x_j) < d_e(x_i, x_k)$ if and only if $1 \leq i < j < k \leq l$, where there is no x_m such that $d_e(x_i, x_m) < d_e(x_i, x_j)$ or $d_e(x_j, x_m) < d_e(x_j, x_k)$, and a monotonic curve in $\boldsymbol{X}$ is described as above by printing all subscripted literals in bold face.

THEOREM 3: If (x_i) is a monotonic curve in X then $\varphi(x_i) = (\boldsymbol{x}_i)$ is a monotonic curve in $\boldsymbol{X}$. Proof directly from Lemma 2.

COROLLARY: Every straight line in X maps on to a monotonic curve in $\boldsymbol{X}$.

With this corollary we have a clue to constructing an appropriate "simplest surface" S in M containing $\boldsymbol{X}$. For let the input frame be described as a $\sqrt{n}$ by $\sqrt{n}$ grid of $2\sqrt{n}$ straight lines at the intersections of which are the frame bits $x_{i,j}$, $l \leq i, j \leq \sqrt{n}$, themselves. Then these grid lines map on to a family of $2\sqrt{n}$ monotonic curves in M whose intersections at $\boldsymbol{x}_{ij}$ preserve the closest-point relationship derived in Theorem 2. It is easy to see that these mapped grid-lines should

not cross an odd number of times except at the x_{ij}, or otherwise become tangled, lest the ordering so necessary for the monotonic property be disturbed. If we imagine a soap film stretched over the interstices of the grid in M a reasonably vivid model of a "simplest surface" containing X results.

If we consider the sequence of sets of points closest to some one x_{ij} in X, next closest, closest but two, and so on, Theorem 2 and Lemma 2 are satisfied if S is a plane or a hemisphere in E^3 but not otherwise. But the closest points-into-closest points relationship must hold for all x_{ij} in X, x_{ij} in X. Then since a sphere cannot be tiled by "squares" whose sides are great circle segments (and equal chords connecting corners to centers, our distances in E^3) it follows necessarily that S must be a plane, and that the monotonic curves of the last corollary are in fact straight lines in S. This plane may be larger or smaller than the input frame; with respect to some fixed reference in E^3 it may be found arbitrarily translated; and of course it can have any rotational orientation about a translated point. The only admissible mappings φ, in other words, are the Euclidean similarity transformations in three dimensions, and no scale changes may occur from one part of the map to another.

With the preceding comments we thoroughly exhaust one-one mappings consistent with C_1 and C_2. These may be succinctly characterized as plane-into-plane, similar, and hence order-preserving in terms of the metrics applicable to image and map. We shall conclude this discussion of the mapping problem by examining some consequences of the question: is the class of allowable mappings augmented by limiting the distance in M over which C_2 is assumed effective? In other words, what are the consequences of assuming that x and y have no effect upon machine organization as pairs (over that which they could be expected to exert merely as two individual elements) if they are more widely separated than some limiting distance r in M? More formally, we modify C_2 as follows:

$$\left.\begin{array}{l} d_e(x, y) \leq d_e(w, z) < r \text{ implies} \\ \\ p_f(x \mid y) \geq p_f(w \mid z), \\ \\ \text{otherwise} \\ \\ p_f(x \mid y) = p_f(w \mid z) \\ \\ = p_f(\alpha, t),\ \alpha = w, x, y, \text{ or } z \end{array}\right\} \quad (C_{2.1})$$

Clearly all that has been said before continues to hold for every subvolume in E^3 containing portions of M, which is less than r in its longest dimension, except that the image of any corresponding portion of X will be bounded by some closed curve c_r in X (perhaps including the boundary of the input frame).

The effect of $C_{2.1}$, in other words, is to localize the restrictions upon φ to regions of X in such a way that portions of the map more than r units away from each other have no direct mutual dependency but are only related by virtue of intervening overlapping regions. But the class of mappings φ remains as before because of the overlapping unless r is impractically small. The adoption of $C_{2.1}$ in place of C_2 implies a corresponding and obvious modification of C_1; by $C_{1.1}$ we shall mean that the original constraint holds only within each c_r. This ends the formal development.

In terms of mechanism $C_{2.1}$ has a certain reasonable appeal: in an actual pattern-learning machine we can readily imagine that $p_f(x \mid y)$ should be a much more sensitive function to closest x, y than even to those twice as widely separated; clearly one of the most important metric properties of useful patterns to which we would desire early establishment of some machine correlate is adjacency of pattern bits. It thus would appear natural that $p_f(x \mid y)$ fall off quite steeply as $d_e(x, y)$ increases from its minimum non-zero value. An exponential or similar functional form would seem appropriate. In such a case, as separation increases there must come a point at which $p_f(x \mid y)$ hardly differs from the sum of the independent effects of $x = 1$ and $y = 1$ separately, and $C_{2.1}$ merely assumes that this distance is significantly smaller than the dimensions of the volume enclosing the machine elements themselves. If r is very much less than these dimensions X is still a plane in M, being built up by connected and overlapping small regions each of which is a portion of a plane.

The preceding derivation of properties of one-one mappings consistent with $C_{1.1}$ and $C_{2.1}$ in an over-idealized machine has consequences of some significance due to the very generality of the assumptions used. This specificity of allowable machine structures and maps really follows from the distance ordering and other properties of metric spaces in three dimensions. We inhabit such a space (E^3 in the small) and our machines exist in it as well. That the mild constraints adopted should necessarily lead to a more than merely "continuous" mapping of a visual field on to a specialized plane surface in a volume of essentially undifferentiated elements is perhaps not intuitively obvious and, since animals have evolved in this same space, might be expected to illuminate certain facts of neuroanatomy whose relationship has been obscure. We will return to this point in the next Section after a short and informal discussion of special one-many "mappings" of X on to M.

Under the assumptions adopted for the derivation of Theorem 1, many-one mappings of any sort are prohibited. In the case of the specialized class of plane surfaces in E^3 an input frame maps by similarity transformation on to one of the planes only. This relationship was derived as a consequence of limiting connections to one-one mapping rather than one-many. We now consider some one-many connection modes consistent with earlier developments.

As a first case let us modify our point of view of the machines previously discussed by regarding the members of X primarily as terminations of connections from X. Then certainly other elements in M are influenced by stimuli arriving at the x's since C_2 or $C_{2.1}$ can reasonably apply to all members of M if we amplify the statement of these constraints to include stimuli from elements in X as well. If other elements in M are much larger in number than the x's (and hence of $N(X)$) we have a quasi one-many "mapping" of X on to M. This would become explicit were C_2 or $C_{2.1}$ to be implemented by specifying that interconnections between members of M (including those of X) tend to fall off monotonically with distance in E^3, the mapping being then on to those elements in M which are not in X but which are connected to elements in X. If the inverse distance function is invariant in form with respect to direction in

E^3 and hence in X we may speak of a unimodal two-dimensional probability distribution of connections from an element x, defined in a plane in M parallel to X (or perhaps containing X) which is maximum closest to x.

A second variety of one-many connections corresponds to adoption of $C_{1,1}$ and $C_{2,1}$ only. Within the volume containing M we may specify more than one plane "simplest surface" as previously discussed, provided no two points on different surfaces lie less than r units of distance from one another. For k such surfaces we have k one-one mappings from X on to X_1, X_2, . . ., X_k in M or, equivalently one one-many "mapping" from X on to $X = \bigcup_{i=1}^{k} X_i$. Combinations of these two types of mappings offer a third, mixed category.

Similarity transformations do not permit changes of scale on the map. Hence if it is desired to obtain finer resolution in the center of the input frame than at the periphery, as would be appropriate if the input frame were to scan the visual field instead of passively receiving information from it, we must effect a preliminary transformation ψ of the visual field on to the input frame. Two general kinds of techniques can be conceived to accomplish this. A nonlinear optical system might focus the visual field on to the input frame giving the desired enhancement at the center. Here the formal statements of C_1 and $C_{1,1}$ remain unaltered but refer to a visual field distorted by ψ^{-1}.

The second solution would crowd the input frame bits to a compact cluster in the center and space them more widely toward the edges or, alternatively, effect the same transformation ψ^{-1} at an intermediate relay location between input frame and machine proper. In either of these cases or in both combined the Euclidean metric d_e is no longer applicable to the input frame and C_1 and $C_{1,1}$ must be altered accordingly. In terms of the new metric d^1 all of our previous results continue to hold provided d^1 is substituted for d_e wherever the latter appears. The visual field then maps on to X by the transformation $\psi\varphi$ as a moment's reflection will show is also the case for the optical solution. Geodesics in the map generated by ψ on the input frame will in general no longer be straight lines in E^2, although special solutions may exist where this is true. It is essential in the foregoing that ψ be order-preserving in point triples with respect to d_e and d^1, and it would be desirable for local departures of d^1 from d_e to be minimal so that small areas of the visual field are not too distorted in their representation upon the plane containing X. As we shall see, these considerations suggest a viewpoint from which to examine mammalian visual system neuroanatomy.

V Visual System Neuroanatomy In Mammals

It was suggested in Section IV that since animals have evolved in a metric space, the ordinary Euclidean world of three spatial dimensions, some of our earlier observations may cast light on hitherto obscure relationships between facts known to neurophysiologists and neuroanatomists. Certainly we would not expect clear contradictions between such facts and the generalized model under discussion if common features can be found characteristic of both.

Constraint $C_{1.1}$ is reasonable to assume as important in early visual learning in mammals. Considerable data exist to show catastrophic consequences of early deprivation of visual experience, notably von Senden's work with cataract patients who received their sight for the first time as adults (Hebb 1949). These people required many months, on the average, of patient coaching in order to learn to recognize simple geometric figures in various orientations. This research is significant in view of the fact that formation of myelin sheaths on axons in the visual (striate) cortex, primarily those in the outer pial surface and the inner white matter, occurs in humans early in infancy followed by myelination in the surrounding peristriate cortex (Conel; Flechsig). Dendritic growth and ramification in the human cerebral cortex is especially rapid in the weeks immediately following birth and again occurs in the striate area earlier than in the parastriate (Conel). In the primate occipital lobes Von Bonin, Garol, and McCulloch have shown (1942) by strychninization techniques that the striate transmits stimuli to the parastriate cortex only, except for slight edge effects at the boundary between these two areas, and that the striate cortex in turn is the

only occipital region to show prompt time-locked evoked potentials in response to light flashed in the eyes of the anesthetized animal. There is a paucity of association fibers in the visual cortex, there being no association fibers which are found to extend more than five millimeters from a lesion to adjacent parts and no further elsewhere than to the parastriate area (Clark). The layout of the striate area thus emphasizes formation of associational connections between regions within it which are close to one another. These studies together indicate that the parastriate functionally builds upon the more fundamental striate area. The formation of myelin insulating sheaths is thought to have something to do with increasingly specific neural organization, and growth of dendrite trees certainly does. It appears that if elementary visual learning is not accomplished at the proper time in a human organism's growth cycle it can be done later with great difficulty if it can be done at all. The possible role of optic tissue degeneration in humans is unfortunately obscure; no anatomical studies have been made on congenitally blind infants (Mendelson). Other studies have shown that cats and chimpanzees reared in darkness grow up to be quite disorganized animals (Riesen). While these data may not definitely imply some physiological and behavioral analogue of C_1 or $C_{1.1}$ the evidence seems all to be in that direction.

Concerning $C_{2.1}$, the English neuroanatomist D. A. Sholl has published findings (1953) on connection pathways in the cat striate cortex which show a marked exponential falloff with distance from the perikaryon (neuron minus axon and dendrite fibers) of occurrence of a neuron's dendrites, the local connections in the cortex apparently being otherwise random. Haddara has since found (1955) a similar situation in the mouse. Uttley (1955) used Sholl's data to show that probability of two neurons being connected together via a synaptic junction falls exponentially with increasing separation between them. Bullock has recently (1959) published evidence of decrementally-spreading interneuron influences in simple ganglia in the lobster, propagating in a continuously decreasing manner with distance and without axon spike generation, and hypothesizes that such continuous and graded activity may be important in cortical functioning. The branchings of afferent nerve terminations in the middle layers of the cat striate cortex occur after the afferent loses its insulating myelin sheath, which it carried while entering through the lower (inner) layers, and show the same dichotomous structure in Sholl's diagrams (1956) as for dendritic ramifications of pyramidal and stellate neurons occurring in the same layers and to which exponential falloff applies, although he does not remark on this similarity. Sholl was careful to restrict his observations and measurements to flat regions of the cat striate cortex since the effect of curvature in the convolutions was not clearly understood.

More recently Bok (1959) in the Netherlands has produced evidence to support his own contention that nerve cell shape and connection patterns are modified in the sulci and gyri in a way appropriate to nullify the effects of cortical curvature in these regions, and states his belief that the cortex is essentially a flat organ. That such alterations to counteract effects of curvature are functional and not accidental or due merely to mechanical

crumpling is persuasively argued by Bok's further observations that macroglial cells and capillary patterns remain unaltered from flat to curved cortical regions; their presumed respective functions of providing a mechanical skeleton and draining fluid require a constant volumetric density distribution in the cortex.

Visual pathways in the mammal form a highly complex system. The retina itself must be regarded as an extension of the brain (Hartline; Young) and much early processing of incoming information occurs there, such as sharpening of contrasts, emphasizing changes with time, and some accommodation to average intensity. After leaving the eyeball the optic nerve bundle terminates in the lateral geniculate body of the thalamus from which cortical afferent nerve fibers radiate to the striate cortex, terminating there (in the cat) in dichotomous branchings in the middle third of its thickness and distributed over its surface at a presumably even density of about 25,000 fibers per mm^2 (Sholl). Since the striate region is at once the thinnest (1–2 mm in cats, about 1.6 mm in man) and most regular part of the entire cerebral cortex it is clear that the afferent endings lie in a surface very thin in comparison to its area (about 3,000 mm^2 in man (Sholl)), and from the work of Bok and his colleagues, in a surface which functionally may be a plane.

There is no one-one mapping of nerve connections from retinal input cells on to striate cortex. Yet point light stimuli in the left or right half of the visual field seen by either eye of the mammal map one-one on the contralateral (right or left respectively) occipital lobe, the site of the visual cortex, in terms of the center of the region of maximum evoked electrical response. This response field for a fixed point light stimulus falls off rapidly as the detecting electrode is moved away from the maximum spot. Visual maps plotted by this technique as well as by nerve degeneration studies (Krieg, Polyak) show disproportionately large areas of the cortex serving foveal vision with compression towards the edges of the striate area as the stimulus is moved toward the visual periphery (Hebb, 1949). In the chimpanzee this "eye map" upon each occipital lobe is quite accurately expressed as a distortion of a semi-polar grid to a family of semi-ellipses compressing logarithmically away from the central (foveal) region, which itself is larger than implied by the logarithmic relation alone (Marshall, Woolsey, and Bard). These mappings are more than merely topologically invariant. We do not observe widely separated points of the visual field mapping on to points of the cortex closer together than the maps of intermediate points as often occurs, for example, in boundary regions of the continuous conformal mappings of function theory, always excepting the central split into two maps located on the two cerebral hemispheres. They are order-preserving with respect to triples in terms of the geodesics of image and map, as are all continuous mappings on metric spaces, but the extreme distortions allowed by continuity alone are not found in mammals, presumably because the striate cortex is in a sense a finite point set in E^3, the lower limit to neuron spacing being set by the size of their perikarya. The maps appear to be an excellent natural compromise between the organism's conflicting requirements of expanded central vision, necessarily implying scale change, and fairly small local deviation from the

Euclidean metric of the world it inhabits.

We have seen that a reasonably good case can be made for correlates in mammals of the constraints $C_{1.1}$ and $C_{2.1}$, and have noted the nature of the order-preserving visual mappings found in these animals and in man. In view of Nature's usual parsimony it would seem that some functional reason of great value to the organism must exist for such highly specific mapping of visual data on to a sheet of randomly but decrementally connected neurons, and it is suggested that this mode of mapping as well as the planar form of the visual cortex itself was forced upon the evolutionary process by the prior existence of similar nerve cells such as found in lower organisms (analogous to $C_{2.1}$), by the properties of Euclidean space, and by the evolving mammal's need to organize its perceptual data on a foundation suited to that space and to the objects in it (analogous to $C_{1.1}$) which the animal had to learn to recognize swiftly and surely in order to survive in competition with organisms possessing less flexible nervous systems. Unfortunately this suggestion, inferred from the development in Section IV, is not open to objective verification, although it seems phylogenetically persuasive.

In that development it was pointed out that change in map scale should not be obtained by varying the density of connections over the surface of machine elements X, and that any local scale changes had to be made by some prior transformation before $C_{2.1}$ with its dependence upon d_c in E^3 entered the picture. Insofar as the analogy holds for mammals this transformation appears to be performed in the retina and perhaps in subcortical structures, the latter also generating extra fibers to spread evenly over the striate area. In man there are about 1,010,000 fibers in the optic nerve (Sholl, 1956) which on the certainly erroneous assumption of one-for-one synaptic relaying in the lateral geniculate body would yield the unreasonably low average density of about 300 per mm^2 at the area striata. The corresponding figure for the cat is 25,000 per mm^2 measured by Sholl for a section of visual cortex beneath a plane pial surface (1956). Even though cortical neurons are more widely separated in man than in cat (Bok; Sholl, 1956) this discrepancy is too great to be tolerated. Cross-species comparisons of this kind are not without an element of risk, however. Clark states that in the monkey 1350 geniculate cells project on to each square millimeter of visual cortex (1942). Yet examination of photomicrographs of sections of striate cortex in cat, monkey, and man does not suggest widely varying ratios of volumes of combined afferent and efferent axons (white matter) to volume of visual cortex served by them (Sholl, 1956; Von Bonin and Bailey; Von Bonin, McCulloch, and Bailey), and hence it does appear that there are many more fibers in the optic radiations than in the optic nerve.

An interesting corroboration of the present theory would result if subsequent neuroanatomical studies were to show that density of input afferents (the optic radiations) is indeed constant or very nearly so over all of the striate cortex, correcting for curvature by converting to cartesian form gaussian coordinates as used by Bok to metrize his cortical sections.

Returning to the continuous mapping of visual field on to area striata, the burden of our discussion is that metric attributes of visual objects are enhanced thereby and it has been suggested that a distinct sur-

vival value is served by this enhancement. The arguments in Sections II and III by analogy do not favor formation of such spatially highly specific nerve paths on the basis of a mammal's short visual experience during early life; it is much more probable that all of evolutionary time was needed for their specification. Accordingly, we may predict that visual mapping studies on mammals reared from birth in light-free and pattern-vision-free environments will show evidence of continuous maps on the striate area. But it is also reasonable to expect that such maps will be diffuse in fine structure when compared with those of normal animals, who have had the advantage of early visual experience in our Euclidean space to flesh out the skeleton, as it were, of their common genetic endowment. Similar remarks would apply to mammals whose early visual experience is not of a metric space at all but of randomly changing spots as on a detuned television screen. Such diffusely lighted or flickering environments would preserve monkeys from degeneration of retinal ganglion cells and in the latter case exercise "on-off" optic functions as well. Retinal degeneration has never been encountered in cats raised in complete darkness, however, even after a three year confinement (Riesen, 1960).

Studies are planned for the coming year to check some of the hypotheses discussed in this Section by appropriate experimental techniques. But in view of the available evidence it would seem almost certain that a major function of the area striata is the facilitation and formation of sensory-sensory associations between contemporaneous elementary stimuli from the visual field which are spatially contiguous or quite close together, and continuous, order-preserving maps are a necessary precondition for this. The basic integration of such contiguous stimuli into the elementary perception of straight lines as a result of eye movement in the presence of a visual field containing point light contrasts has, for example, been extensively analyzed by Hebb (1949).

Cortico-cortical connections within the striate area are significantly local. Association fibers leaving the monkey visual cortex re-enter it immediately, most of them within 2 millimeters and decrementally distributed as required by our theory, and except for those destined for the parastriate or the superior colliculus all have re-entered within 5 millimeters (Clark; Polyak, pp. 435–436). Braitenberg has described similar but intra-cortical horizontal local myelinated connections in the region of Gennari's stripe, roughly the middle-third layer. Continuous or topological mappings are defined as those which map all small regions into corresponding small regions. That such an operation necessarily generates an often distorted but usually recognizable map of the original figure, in much the same way that successive joining of individual links eventually produces a chain, is in this case merely a neuroanatomical accident due to the mathematical fact that Euclidean space is also a topological space (as are all metric spaces). Except in the case of Hebb, whose theory nicely accommodates it, this accidental consequence seems to have confused our view of the neurology of visual perception for a generation.

VI Discussion and Summary [29]

All of the mechanisms discussed in this paper are essentially static. Where time changes have been introduced, as in the concepts of $f(x, t)$, $p_f(x \mid y)$, and C_1 and C_2 in Section IV, rather slow changes "clocked" by $t = 1, 2, 3, \ldots$, the sequence of pattern learning trials, are implied. We have not introduced any reverberatory, scanning, or noise-like mechanisms although analogues to these may well exist in living organisms and certainly are often encountered in pattern recognition schemes programmed upon digital computers. For example, Hubel has found exceptions to the simple visual map in the cat when more or less rapid motion is taken into account. Some single neurons in the cat striate cortex fire only when a point light source is moved across the visual field but not when it is merely blinked on and off, while others only cease firing under such conditions; and still others are sensitive to movement of large objects across the field. Whether such effects are due to genetically specified anatomy alone or need early-life visual experience to become evident might be clarified by experiments with animals reared in darkness. Such special temporal activity is not inconsistent with our previous developments; it is merely that the viewpoint of this paper relegates them to the category of means or does not encompass them. The slow changes referred to earlier are intended to be more in line with Hebb's postulate, applicable in principle to mechanical (machine elements *) as well as to biological neurons: "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one

of the cells firing B, is increased." (Hebb, p. 62). This assumption is basic to the formation and stabilization of Hebb's cell assemblies and phase sequences, concepts which have had a profound influence upon modern behavioral studies and indeed which have been mathematically shown by Beurle to be applicable as well to a sheet of highly stylized neuron-like elements obeying the exponentially falling-off connectivity mode of Sholl and Uttley.

The attempt has, rather, been made to arrive at a most general description of pattern-learning devices by consideration of principles which must hold for all such machines. One such principle is that completely random "organization" is not practical because any sequence of equisignificant patterns of reasonable length is simultaneously a subclass of an impossibly large number of abstract pattern (or rather, frame) classes only a small fraction of which is desired. Implications of this important point have previously been overlooked with few exceptions (Day and Newman; Kalin; Beer).

A second principle is that the kinds of visual patterns that interest us are built up from fundamental ordering properties of Euclidean space. Indeed, for much of our analysis a space with completely ordered distances and without an overt metric would have sufficed. A third principle follows from the previous two: some sort of structure must be imposed upon initial machine chaos to enable an automaton to emphasize these ordered properties of patterns. While almost all workable pattern recognition devices exhibit a highly sophisticated structure (for example, Bledsoe and Browning; Roberts; Doyle) we have been content to state two quite mild constraints upon initial randomness and rigorously to exhaust their consequences.

It has turned out that these mild constraints proved sufficient to couple our devices effectively to Euclidean space, the properties of which were shown to require mapping of an input frame on to a plane with no local scale change. This unexpectedly specific conclusion was derived independently of any particular inverse functional dependence of machine organization change with distance of separation of simultaneously stimulated pattern bits, provided only that proper monotonicity be preserved. In retrospect it is easy to see that a certain controlled amount of warping and stretching of the map would have been permitted by addition of some suitably small numerical tolerance $\pm \epsilon$ to the antecedents of the implications C_1 and $C_{1.1}$ in Section IV, closest points in X then mapping inside an annulus in X of thickness $2k\epsilon$, k the dilation of φ, but there seems to be no real need for this additional complexity; indeed Sholl's data on exponential fall-off of intracortical connections implies the contrary (1956, Fig. 8). The inclusion of this artifice in the formal development would complicate the proofs as the price of the greater generality obtained but in turn might make the analogy between our formal development and mammalian visual system anatomy more palatable.

As it is this analogy seems intriguing enough. It connects the hitherto unrelated facts of topologically invariant visual maps and locally random but exponentially falling off connection modes in the interior of the mammalian visual cortex in terms of one theory which has a plausible foundation in evolution. Indeed, as we ascend the phylogenetic scale through

rabbit, cat and primate to man, we find the primary visual center progressively displaced from the tectum in the old brain, increasing lamination of the lateral geniculate relay nucleus as both visual fields progressively overlap, and increasing focussing of optic radiations from that nucleus on to the area striata of the cerebral cortex until in man that is their only, sharply defined, terminal site (Bishop, Burke, Davis, and Hayhow).

The principles underlying the theory are clearly applicable to other cortical sensory projection areas where ordering is important. The dog middle ectosylvian auditory cortex, for example, is functionally laid out in a sequence of side-by-side parallel strips each responding to about 0.1 octave with no spatial reference along the strip to sound from the contralateral ear. This second dimension is correlated with sound intensity of the same frequency from the ipsilateral ear, however, so that a point on the two-dimensional area corresponds not only to sound frequency but to intensity ratio as sensed by the two ears (Tunturi).

Of course the actual neurological mechanisms involved in hearing and sight are incredibly more complex than the present naive model would suggest. No real mention has been made of binocular vision and of what role the delicately sculptured lamina in the lateral geniculate body with its precise geometry might play in this connection, nor about occulomotor feedback paths and their possible function in pattern learning, nor has more than passing reference been made to further elaboration upon fundamentally ordered data from the visual field so clearly necessary to effective recognition of instances of abstract visual forms by machine or mammal. And, lastly, we can only allude to the primal mystery of visual perception — how all of these diverse but coordinated neurological phenomena somehow coalesce in the unified conscious experience of seeing our world.

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AFFERENT or AFFERENT FIBER
An axon originating elsewhere entering a region of nervous tissue upon which it exerts an effect.

AXON
That portion of a nerve cell, or neuron, whose main function is the propagation of an all-or-none discharge from the main cell body to distant points. Axons may be quite long — up to three feet in the peripheral nervous system — but ordinarily extend a few millimeters or centimeters in the cerebral cortex. "White matter" consists almost exclusively of myelinated axon fibers. Antidromic or backward firing of axons is easily induced in the laboratory but probably occurs only rarely in living animals, and then only in the peripheral nervous system.

BINARY QUANTIZATION, See BIT

BIT
Contraction of Binary Digit. The binary digit, 0 or 1, is the elemental unit of information in modern communication and information theory (Shannon) and is featured prominently in this paper because of close analogies to inclusion or exclusion of an element in a set or class, to combinatorial operations on integers, and to the "on-off" mode of operation of modern digital electronic computers. A binary number is a positional sequence of binary digits weighted by successive powers of two; thus any integer $I = \sum_{0}^{n-1} b_i 2^i$, $0 \leq I \leq 2^n$, and any algebraic number $N = \sum_{-\infty}^{\infty} b_i 2^i$ where the b_i's are binary digits. These simple relationships allow us to "quantize" lengths, magnitudes,

threshold values, etc., to a reasonable degree of fineness in terms of an interval containing discrete jumps and thus to express any point within the interval in terms of the step on which it lies, which in turn is typified by a binary number or equivalently by a sequence of binary digits. To avoid constant repetition "bit" is also used in this paper to denote an element of an input frame as well as the binary quantized value of that element, either 0 or 1, a two-step quantization only.

COMPLETE ORDERING

In the abstract, a requirement that a set of elements $\{a\}$ be susceptible to relational predicates $<$, $=$, in such a way that one and only one of $a < b$, $a = b$, or $b < a$ is true for all a, b in $\{a\}$, that if $a < b$ and $b < c$ then $a < c$, and further, that if $a = b$ then $b = a$. $\{a\}$ is then said to be a completely ordered set or space. When these predicates are conventionally interpreted as "is less than" and "is equal to" applied to real numbers it is easy to see that a finite metric space $\{x\}$ of n elements is always associated with a completely ordered set $\{d(x, y)\}$ of $\frac{1}{2}n(n-1)$ elements, the distances between pairs. If such complete ordering cannot be accomplished then of course the corresponding point set is not metrizable. In any case a completely ordered set does not correspond uniquely to some one metric space since it embodies a more general notion than quantifiable distance. The basic arguments in Section IV (Lemma 2, Theorems 2 and 3) are based upon complete distance ordering. "Ordering" in this report means "complete ordering" since we do not work with partially-ordered sets, those for which "$=$" does not apply.

CORTEX

Same as cerebral cortex in this paper. From the Latin for "bark"; the outer few millimeters of the cerebral hemispheres in mammals. Cortical tissue is "grey matter" from the characteristic appearance of unmyelinated nerve protoplasm. Frogs and reptiles, as well as birds, do not have a cerebral cortex or, more properly, a neopallium. Except for the sensory and motor projection areas, and the left temporal lobe "speech center" in most cases, the human cortex shows a puzzling lack of correlation of function with structure (Hebb). Decrease in functional efficiency seems to depend primarily upon amount of non-specific cortical material removed. This is quite opposed to the more genetically specific functional connections in the brains of lower phyla which, on the other hand, exhibit more rudimentary learning activity. Damage to the visual cortex, on the other hand, results in a permanent localized visual defect. Polyak's case "Mallory" is a detailed thirty-year clinical study of such a lesion.

DENDRITES

Branching processes extending from the perikaryon of a neuron whose main function seems to be integration of impulses received through synapses from the axons of other neurons. Recent work suggests that some dendrites may never undergo explosive "all or none" firing activity, however (Bullock). In the central nervous system dendrites may account for as much as nine-tenths of total neuron volume.

EFFERENT or EFFERENT FIBER

An axon of a neuron within a region of nervous tissue but leaving it to produce an effect elsewhere.

EQUISIGNIFICANCE
Meaning the same thing. The basic problem in pattern recognition is the classification of patterns into classes the members of which have the same meaning, as Selfridge pointed out in 1955. Reichenbach employed a similar notion in defining a "symbol" as an equisignificance class of "tokens," or particular physical signs (Reichenbach, p. 4). In introducing this notion in Section II, I have emphasized that two or more patterns of the same class are to have the same meaning to the machine's designers and operators in order to stress the pragmatic aspects of the concept. To say that a machine can handle meanings directly is gratuitous; it is more to the point to recognize that machines deal with physical states and events which symbolize meanings for us. It is then easier to appreciate that other physical entities which the machine might handle equally well may have no meaning — for example, most frames in the abstract classes discussed in Section III.

EUCLIDEAN METRIC
As used in Section IV and elsewhere in this paper, $d_e(x, y) = ((x_1 - y_1)^2 + (x_2 - y_2)^2)^{1/2}$ where $x = (x_1, x_2)$, $y = (y_1, y_2)$ in cartesian coordinates, and $d_e(\boldsymbol{x}, \boldsymbol{y}) = ((\boldsymbol{x}_1 - \boldsymbol{y}_1)^2 + (\boldsymbol{x}_2 - \boldsymbol{y}_2)^2 + (\boldsymbol{x}_3 - \boldsymbol{y}_3)^2)^{1/2}$ also in cartesian coordinates. The two sets of points X and $\boldsymbol{X}$ are respectively embedded in the infinite Euclidean spaces E^2 and E^3, metrics on which of course remain invariant in value for given points with respect to any changes of coordinate system adopted but assume in general different forms of expression for different coordinate schemes. This is the metric peculiar to our immediate spatial world in which instances of visual patterns are found as well as machines for recognizing them, and of course our own central nervous systems have evolved and exist in Euclidean space as well. In this world E^2 is included in E^3 and is not a separate entity, as is often convenient to assume in the abstract. It is in this sense that we speak in Section IV of some "specialized plane" in a volume of machine elements and of the cortical surface evolving in E^3, in Section V.

FRAME
Any one of the 2^n possible configurations of binary digits corresponding to the binary quantizing modes of an input frame.

GANGLION CELLS
In this paper, retinal ganglion cells. These are neurons in the retina whose axons form the fibers of the optic nerve.

GEODESIC
A curve in a metric space comprised only of shortest distances. For all points x, y, z in that order along such a curve, $d(x, y) + d(y, z) = d(x, z)$.

INPUT FRAME
In the case of visual pattern recognizing machines, a finite two-dimensional array of quantizing optical transducers.

LATERAL GENICULATE BODY
Syn.: Lateral geniculate relay nucleus, lateral geniculate nucleus. One of two subcortical structures each receiving optic fibers from the retinas of both eyes, corresponding to half of the visual field, and sending fibers to the striate area of its hemisphere. A laminar structure becomes increasingly evident as the phylogenetic scale is ascended, optic fibers from the two

eyes ending in alternate laminations which in the cat have some common neurons at the interfaces but not in primates and man. The function of this body seems intimately related to binocular vision and perhaps to color vision in primates and man (Clark). From the viewpoint of the present paper additional functions may be participation in the transformation ψ^{-1} of Section IV and generation of extra fibers as suggested, for example, by the microscopic geometry of the cat geniculate body (Fig. 1 of Bishop, Burke, Davis, and Hayhow, and Fig. 216 of Polyak), of monkey (Fig. 211, Polyak), and of man (Fig. 213, Polyak).

METRIC

A distance measure on a set. A metric $d(x, y)$ on a set of elements $\{x\}$ specifies a real non-negative number, the "distance" from x to y. This number is zero if and only if x and y coincide, and is the same from y to x as from x to y. Distance means shortest distance: all triples x, y, z in $\{x\}$ satisfy the "triangle inequality" $d(x, y) + d(y, z) \geq d(x, z)$. The set $\{x\}$ is then said to be a metric space. (Busemann; Hilbert and Cohn-Vossen). Euclidean space is an example, as is the hyperbolic space of Lobatchevskian geometry. The trivial metric space defined by $d'(x, y)$ in Section III is an instance of Busemann's metric space R_1. The finite metric spaces considered in this paper refer directly or indirectly to an input frame upon which a pattern is quantized into n bits, and never to a space of 2^n points the elements of which stand for possible frames. In other words we have not ordered, let alone metrized, any class of equisignificant patterns. Such a class is a subset of the space of 2^n frames and can have elements from many distinct pattern classes as members simultaneously as was argued in Section III. An observer confronted with this situation would no doubt prefer to order these elements differently with respect to their resemblance to some one "archetypical" pattern in each of the overlapping equisignificance classes. Thus the subset would not be metrizable as such since no unambiguous ordering of distances would be available. It is possible, on the other hand, to order adequately the patterns within one equisignificance class with respect to an archetype of that class if all other classes are ignored. It has also proved possible in practice to identify an unknown pattern lying in one of a limited number of largely mutually-exclusive equisignificance classes with its proper class by measuring its distance (suitably defined) from each of the several archetypes in turn and choosing the smallest number (Bledsoe and Browning).

OPTIC RADIATIONS

The physiological term used to describe axons from neurons in the lateral geniculate body of the thalamus that terminate in the visual cortex. Distinguished from "optic fibers" of the optic nerve which enter the geniculate body from the retina and initiate activity in the optic radiations by synaptic connections in the geniculate layers.

PATTERN

A humanly-meaningful frame. This terse definition merely reflects the truism that we are only concerned with machines which interact with the physical world in a way interesting or useful to us.

PERIKARYON

Syn.: Main cell body. A neuron minus its dendrites and axon(s). That part of the nerve cell containing the nucleus, and the site of significant metabolic functions (Eccles). In the cerebral sensory cortex activity in the neighborhood of its associated dendrites is the main factor in determining whether an all-or-none discharge occurs in the perikaryon (and hence propagated down the axon), but multiple synaptic junctions directly on the perikaryon are typical of spinal motor neurons.

PROBABILITY

Probabilistic notions used in this report have purposely been vaguely phrased to preserve generality. In Section III $p_f(x, t)$ would, strictly speaking, adequately refer to a probability function only if some random-like process were involved — so that repeated runs of the machine with the same sequence of patterns as $t = 1, 2, 3, \ldots$ converge upon $p_f(x, t)$ at the t^{th} trial, $t \to \infty$. If the machine is determinate, with no internal noise sources, such convergence cannot occur because the sequences of machine states will be identical under identical conditions, a finite-state automaton in the conventional sense, and $p_f(x, t)$ must then only refer to our lack of complete knowledge as to how the device is wired internally. In this case we would require the machine to be so internally organized that the constraints C_1, C_2 hold in a sense comparable to that characterizing one sequence of trials on the internal noise source device. Analytical methods applicable to describing the behavior of determinate machines may vary from those derived from the theory of finite-state automata (Kleene; Minsky) to complicated formulations involving continuously ranging variables. The internal-random-process machines may be characterized by finite Markov chains or discrete Markov processes depending upon whether transitional probabilities between states depend upon the value of t (Kemeny and Snell). Although it is perhaps difficult to conceive of a learning machine whose next state depends uniquely upon its present one, digital computers upon which so many pattern-learning schemes have been simulated fall into this category unless they include a noise source or a random number table inaccessible to the programmer. In any event it is reiterated that no commitment to any of these various alternative positions has been made in the text (Sections IV and VI).

SIMILARITY TRANSFORMATION

A transformation group in the Euclidean plane characterized by translation, rotation, dilation, and reflection. If we associate a complex number with each point in the Euclidean plane the similarity transformations are conformal mappings $w = az + b$ or $w = a\bar{z} + b$ (with reflection), $\bar{z}$ the complex conjugate of z, a and b complex, and $a \neq 0$. (These are the only conformal mappings carrying E^2 into itself). The dilation and rotation of such a mapping are specified by a, and translation by b. The plane-into-plane similarity transformations in three dimensions, the φ discussed in Section IV, refer to this two dimensional group plus additional data needed in any given case to specify the relative position and orientation in E^3 of S and the input frame, but as a class do not themselves technically form a group unless both X and $\boldsymbol{X}$ are considered embedded in the abstract Euclidean plane.

SYNAPSE
A junction between axon of one neuron and dendrite of another, over which all-or-none spike discharge activity may travel. Some kind of summing and threshold function is commonly attributed to synaptic junctions in the central nervous system in order to obtain non-linear discrimination between varying numbers of impulses arriving at one dendritic tree. In spinal motor neurons the important synapses occur directly on the perikaryon.

TECTUM
Syn.: Colliculus. A subcortical region phylogenetically very old. In frogs, reptiles, and birds the primary visual projections are found here but in man they have been entirely displaced to the visual cortex.

TOPOLOGICAL MAPPING
A transformation on a set or space which carries neighborhoods of each point into neighborhoods of uniquely corresponding points. In the classical topology of infinite sets a neighborhood is often defined as any open set containing the point in question, explicitly allowing such sets to be very "small." But often in such topological spaces there is no notion (even undefined) of "distance," so that complete distance ordering is difficult to tie down. Further, specialization of the most abstract topological spaces to the finite set case requires some supporting machinery irrelevant to our present purposes. The approach adopted in this report has the virtue of requiring a minimum of preliminary mathematical argument, made possible because we assume that the Euclidean spaces E^2 and E^3 (which are also topological spaces) are unavoidable in actual pattern-learning mechanisms, whether machines or mammalian visual systems. We avoid overt commitment to the Euclidean metric in the earlier stages of the argument, however, by tacitly associating with each of E^2 and E^3 a related set of completely ordered distances between pairs of points, as described under **COMPLETE ORDERING**, and by couching most of the arguments in Section IV in terms of inequalities or equalities between such distances. We thus make use of some of the topological properties of Euclidean space without becoming involved with topology as such.

VISUAL CORTEX
Syn.: Striate Area, Area Striata. Seat of the visual primary cortical projection area in mammals. The term "striate" derives from the fact that the region is the most regular as well as the most clearly laminated in microscopic section of all cortical areas. It is located at the rear of the brain and, in man, is partially concealed within the fissure separating the two cerebral hemispheres. The visual cortex occurs in two distinct parts, one in each hemisphere, each of which receives information from one-half of the visual field but originating in both eyes, the signals mixing in the lateral geniculate body. The division between halves is a vertical line at the center of vision. The striate area receives (in man) fibers only from the lateral geniculate body and sends axons only to the surrounding parastriate area, of all other cortical regions, although subcortical efferents to the superior colliculus facilitate occulomotor activity. Electrical stimulation by a fine electrode implanted in the optic radiations (Walter) as well as by probe touching the striate

cortex (Krieg) gives rise to punctuate light sensation localized in space, in the conscious human subject, while similar external stimulation on the parastriate has been reported to cause perceptions of forms without spatial orientation and lesions there dramatically impair pattern recognition performance (Krieg). The subjective experience of visual fusion of both halves of the visual field in normal subjects is apparently facilitated by cross-connecting fibers joining the parastriate areas in opposite hemispheres as well as callosal connections (those between cerebral hemispheres) joining higher areas; there are no such cross connections between striate regions. Striate and parastriate correspond to areas 17 and 18 in Brodmann's architectonic scheme.

X
The set of elements x, called "input frame bits," in terms of which a visual pattern or frame is quantized.

$\boldsymbol{X}$
The set of machine elements $\boldsymbol{x}$ connected to the input frame and presumed to have some sort of interaction capability which can change with time as successive patterns are presented to the input frame.

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